J Math Imaging Vis
DOI 10.1007/s10851-008-0067-4

An Improved Hybrid Model for Molecular Image Denoising

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Abstract In this paper an improved hybrid method for removing noise from low SNR molecular images is introduced. The method provides an improvement over the one suggested by Jian Ling and Alan C. Bovik (IEEE Trans. Med. Imaging, 21(4), 2002). The proposed model consists of two stages. The first stage consists of a fourth order PDE and the second stage is a relaxed median filter, which processes the output of fourth order PDE. The model enjoys the benefit of both nonlinear fourth order PDE and relaxed median filter. Apart from the method suggested by Ling and Bovik, the proposed method will not introduce any staircase effect and preserves fine details, sharp corners, curved structures and thin lines. Experiments were done on molecular images (fluorescence microscopic images) and standard test images and the results shows that the proposed model performs better even at higher levels of noise.

Keywords Fourth order PDEs · Hybrid filter · Molecular images · Relaxed median filter

1 Introduction

Image denoising is an active area of interest for image processing researchers for a long period. The use of partial differential equations (PDEs) in image processing has grown significantly over the past years and a large number of PDE based methods have particularly been proposed to tackle the problem of image denoising with a good preservation of edges, and also to explicitly account for intrinsic geometry [1–5]. Before the development of non-linear PDE based methods, the problem of noise reduction in images was treated through linear filtering, in which the image intensity function is convolved with a Gaussian [6]. The main problem with this method is the blurring of image edges. Since the pioneering work of Perona and Malik [7] on anisotropic diffusion there has been a flurry of activity in PDE based denoising techniques. Although the method proposed by Perona and Malik and its variants are much better in denoising images, these methods tend to cause blocky effects in images. This blocky effect is visually unpleasant and the possibility of detecting them as false edges by edge detection algorithms is high. In [8] it is noted that even without noise, *staircasing* effect can arise around smooth edges. Anisotropic diffusion is designed such that smooth areas are diffused faster than less smooth ones and blocky effects will appear in the early stage of diffusion, even though all the blocks will finally merge to form a level image.

Recently many authors proposed hybrid filters for image denoising e.g. [9–11]. In this paper we focused on improving the hybrid filter proposed by *Jiang Ling* and *Alan Bovic* [9] to denoise low SNR molecular images. This filter is a combination of anisotropic diffusion and median filter. The *anisotropic median diffusion* filter produces excellent results when compared with stand-alone anisotropic diffusion or median filter. But it has certain drawbacks. Since the method uses anisotropic diffusion in stage-1, the filter reflects the staircase effects of anisotropic diffusion. The median filter is far from being a perfect filtering method as it removes

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fine details, sharp corners and thin lines [12]. The main reason is that the ordering process destroys any structural and spatial neighborhood information [13].

So to remove these distortions, in the proposed model we replaced the anisotropic diffusion method with non linear fourth order PDE and median filter with more effective relaxed median filter. The fourth order PDEs proposed by *You* and *Kaveh* [14] is used for analysis. As an alternative to median filter we used a version of median filter, *the relaxed median filter* [12]. This filter is obtained by relaxing the order statistic for pixel substitution. By using a relaxed median filter we can preserve more image details than the standard median filter. The proposed method will not introduce any blocky effects in images (as can be seen in anisotropic median filter) and also preserve fine details, sharp corners and thin lines and curved structures better than *anisotropic median filter*. The following section overviews the fourth order PDEs proposed by *You* and *Kaveh*. Section 3 presents the improved hybrid method and discuss its performance with the existing methods, and finally we summarize the contribution of this paper.

2 Fourth Order PDEs

Non-linear fourth order PDEs is a comparatively new approach for effective image denoising. A number of fourth order PDEs have been proposed in recent years for image denoising [6, 14, 15] and [16]. Although discrete implementation of these methods produces impressive results, very little is known about the mathematical properties of the equations themselves. Indeed there are good reasons to consider fourth order equations. First, fourth order linear diffusion dampens oscillations at high frequencies (i.e. noise) much faster than second order diffusion. Second, there is the possibility of having schemes that include effects of curvature (i.e. the second derivatives of the image) in the dynamics, thus creating a richer set of functional behaviors [6].

In the proposed model we used the L^2-*curvature* gradient flow method of *You* and *Kaveh* [14]. The model is shown in (1).

$$\frac{\partial u}{\partial t} = -\nabla^2 \left[c\left(\left| \nabla^2 u \right| \right) \nabla^2 u \right] \tag{1}$$

where $\nabla^2 u$ is the Laplacian of the image u. Since the Laplacian of an image at a pixel is zero if the image is planar in its neighborhood, the PDE attempt to remove noise and preserve edges by approximating an observed image with a piecewise planar image. The desirable diffusion coefficient $c(\cdot)$ should be such that (1) diffuses more in smooth areas and less around less intensity transitions, so that small variations in image intensity such as noise and unwanted texture are smoothed and edges are preserved. Another objective for the selection of $c(\cdot)$ is to incur backward diffusion around intensity transitions so that edges are sharpened, and to assure forward diffusion in smooth areas for noise removal [17]. Here are some of the previously employed diffusivity functions [18]:

A. Linear diffusivity [19]:

$$c(s) = 1. \tag{2}$$

B. Charbonnier diffusivity [20]:

$$c(s) = \frac{1}{\sqrt{1 + \frac{s^2}{k^2}}}. \tag{3}$$

C. Perona–Malik diffusivity [7]:

$$c(s) = \frac{1}{1 + (\frac{s}{k})^2}, \tag{4}$$

$$c(s) = \exp\left[-\left(\frac{s}{k} \right)^2 \right]. \tag{5}$$

D. Weickert diffusivity [21]:

$$c(s) = \begin{cases} 1 & s = 0, \\ 1 - \exp\left(\frac{-3.31488}{(s/k)^8}\right) & s > 0. \end{cases} \tag{6}$$

E. TV diffusivity [22]:

$$c(s) = \frac{1}{s}. \tag{7}$$

F. BFB diffusivity [23]:

$$c(s) = \frac{1}{s^2}. \tag{8}$$

In the implementation of the proposed method we used *Perona-Malik* diffusivity function as shown in (4). The equation (1) was associated with the following energy functional

$$E(u) = \int_\Omega f\left(\left| \nabla^2 u \right| \right) dxdy \tag{9}$$

where Ω is the image support and ∇^2 denotes Laplacian operator. Since $f(|\nabla^2 u|)$ is an increasing function of $|\nabla^2 u|$, its global minimum is at $|\nabla^2 u| = 0$. Consequently, the global minimum of $E(u)$ occurs when

$$|\nabla^2 u| \equiv 0 \quad \text{for all } (x, y) \in \Omega. \tag{10}$$

A planar image obviously satisfies (10) [14], hence is a global minimum of $E(u)$. Planar images are the only global minimum of $E(u)$ if

$$f''(s) \geq 0 \quad \text{for all } s \geq 0 \tag{11}$$

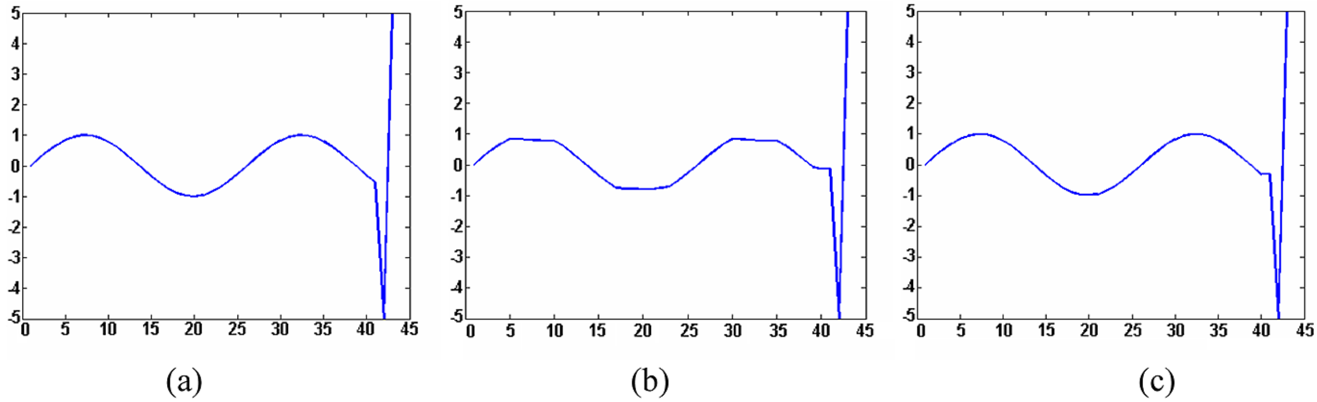

Fig. 1 Figure shows the inability of second order anisotropic median diffusion filter. (**a**) Original Signal. (**b**) Anisotropic median diffusion. It can be seen that blocky effect is introduced in the signal. (**c**) Processed signal by proposed Method. (Here the iteration step T is taken as 3)

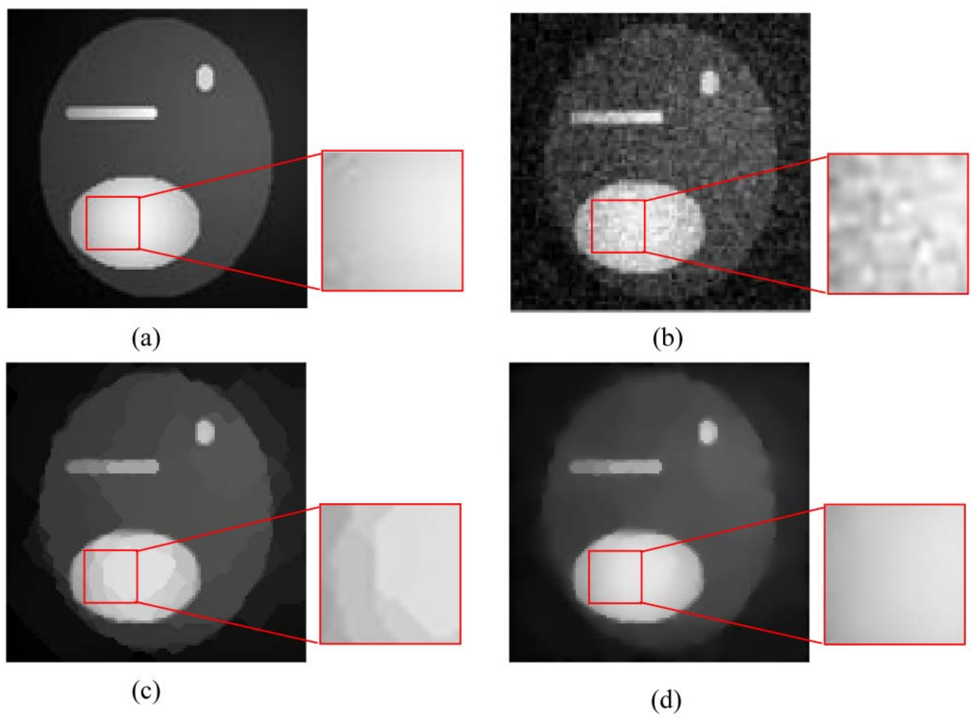

Fig. 2 Low SNR molecular image denoising using improved hybrid filter: (**a**) Phantom cellular molecular image. (**b**) Noisy image (SNR 5 dB). (**c**) Image denoised with Ling-Bovik method. (**d**) Image denoised with proposed method. Close examination shows that Ling-Bovik method generates false edges

because the cost functional $E(u)$ is convex under this condition [14]. Therefore, the evolution of (1) is a process in which the image is smoothed more and more until it becomes a planar image. But in the case of second order anisotropic diffusion $f''(s)$ may not be greater than zero for all s and as a result the image is evolved towards a step image and that is why it suffers from blocky effects. So the image processed by fourth order PDEs will look less blocky than that processed by second order anisotropic diffusion.

Computational experiments shows that the *edge detector* based on this theory yields edges and boundaries that remain more stable through scale t. In spite of this, this model still has several theoretical and practical difficulties. For instance, if the image is very noisy, the gradient ∇u will be very large, and as a result, the function $c(\cdot)$ will be close to zero at almost every point. Consequently, noise will remain when we use the smoothing process introduced by the above model. This can be considerably reduced by attaching a relaxed median filter at the end of the *You-Kaveh* model.

3 Proposed Method

The discrete form of non-linear fourth order PDE described in (1) is as follows:

$$u_{i,j}^{n+1} = u_{i,j}^{n} - \Delta t \nabla^2 g_{i,j}^{n} \tag{12}$$

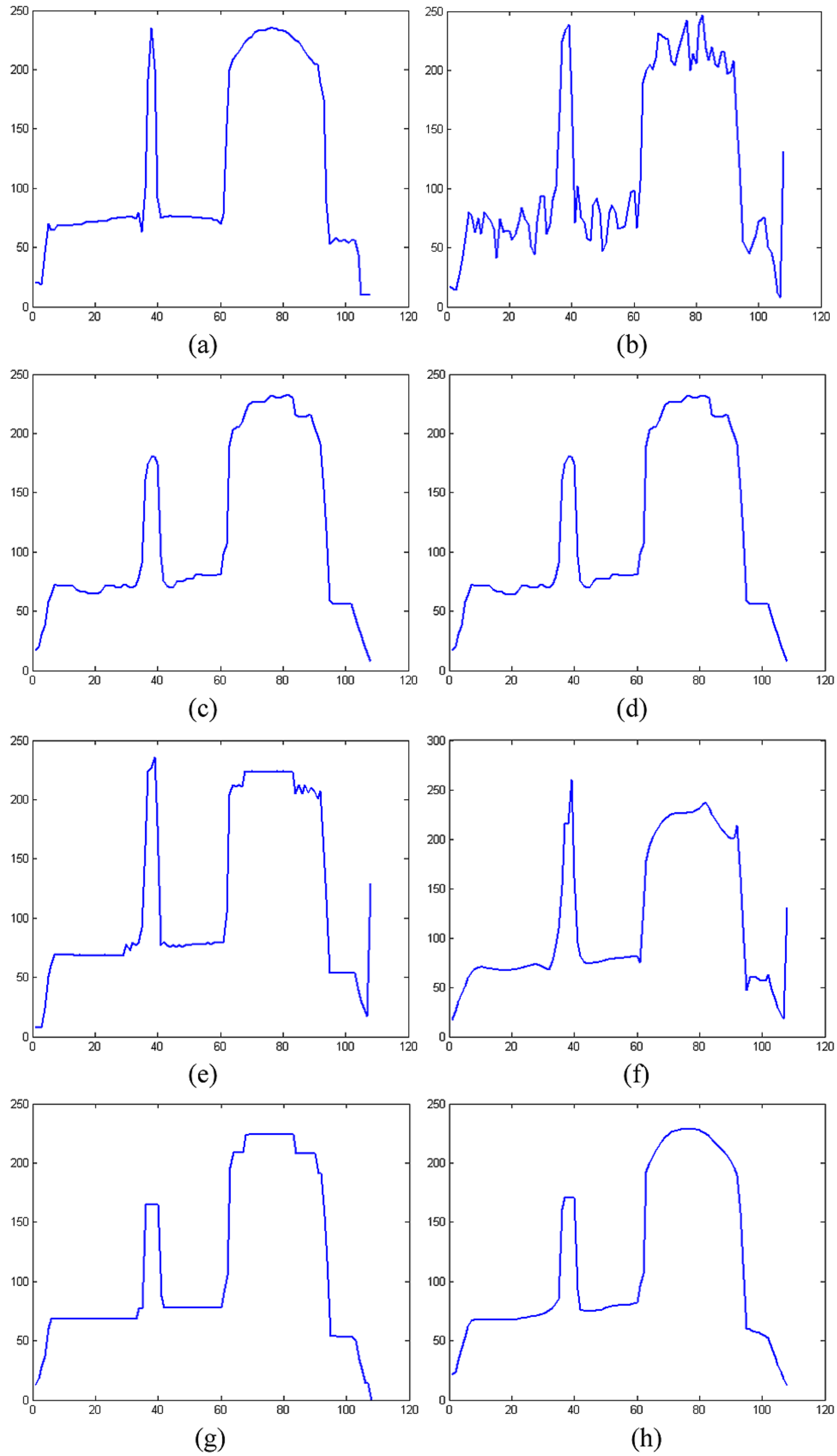

Fig. 3 Image denoising with existing & modified method: (**a**) 54^{th} row of the phantom image (**b**) 54^{th} row of low SNR image, (**c**), (**d**), (**e**), (**f**), (**g**), and (**h**) are the plot of 54^{th} row after processed with iterative median, iterative relaxed median, anisotropic diffusion, 4^{th} order diffusion, anisotropic-median diffusion and proposed method respectively

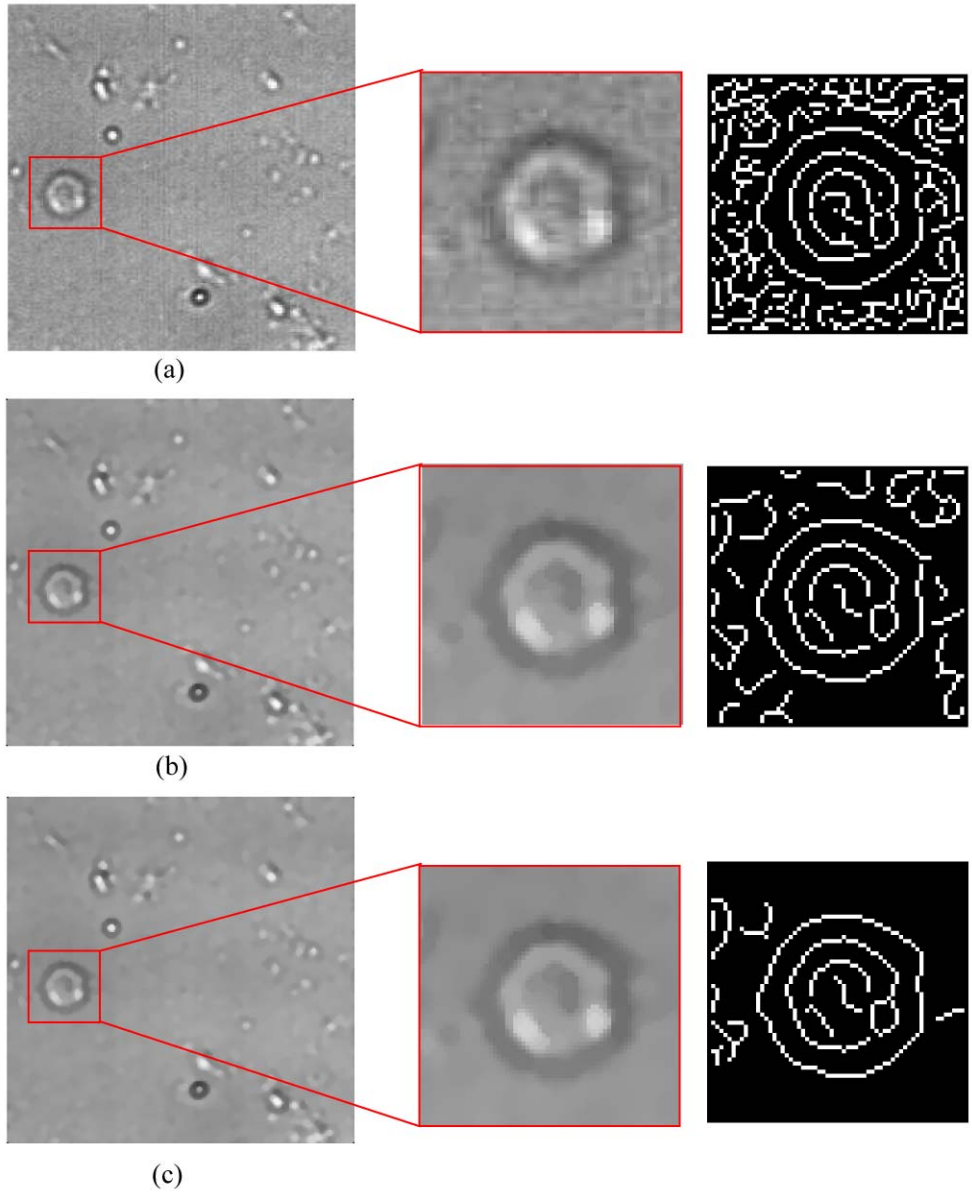

Fig. 4 Endothelial cells of rat aorta processed using anisotropic median diffusion filter and proposed filter. (**a**) Original Image (**b**) Processed with anisotropic median diffusion filter (**c**) Processed with improved method. Visual analysis and edge map shows that the improved method produces better result than existing one

where

$$\nabla^2 g_{i,j}^n = \frac{g_{i+1,j}^n + g_{i-1,j}^n + g_{i,j+1}^n + g_{i,j-1}^n - 4g_{i,j}^n}{h^2}, \tag{13}$$

$$g_{i,j}^n = g\left(\nabla^2 u_{i,j}^n\right), \tag{14}$$

$$\nabla^2 u_{i,j}^n = \frac{u_{i+1,j}^n + u_{i-1,j}^n + u_{i,j+1}^n + u_{i,j-1}^n - 4u_{i,j}^n}{h^2}. \tag{15}$$

Δt is the time step size and h is the space grid size. *You-Kaveh* fourth order diffusion is derived from a variational formulation, much like the second order Total Variation introduced by *Rudin*, *Osher* and *Fatemi* [24]. It is observed in [25] that the *You-Kaveh* equation is linearly ill posed in the regions of high curvature. Behaviour of 2$^{\text{nd}}$ order anisotropic diffusion (Perona-Malik) and 4$^{\text{th}}$ order anisotropic diffusion (You-Kaveh) on a sine wave is shown in Fig. 1. The stair case effect of *Perona-Malik* diffusion (used in *Ling-Bovik* method) can be studied from this experiment.

Relaxed median filter [12, 26] can be used in combination with (1) to remove large spike noises. The proposed hybrid method is defined as follows:

$$u_{i,j}^{n+1} = RM_{\alpha\omega}\left(u_{i,j}^n - \Delta t \nabla^2 g_{i,j}^n\right) \tag{16}$$

where *RM* is the relaxed median filter with lower bound α and upper bound ω.

The working of a relaxed median filter is as follows: it has two bounds, a lower bound (α) and an upper bound (ω), defines a sublist inside the window $[W_{(i,j)}]_{(\cdot)}$, which contains the gray levels that we assume to be good enough not to be filtered. If $Y_{(i,j)}$ is the output of a relaxed median filter, then $Y_{(i,j)}$ can be written as

$$Y_{(i,j)} = RM_{\alpha,\omega}\{W_{(i,j)}\} = \begin{cases} X_{(i,j)} & \text{if } X_{(i,j)} \in [[W_{(i,j)}]_{(\alpha)}[W_{(i,j)}]_{(\omega)}], \\ [W_{(i,j)}]_{(m)} & \text{otherwise} \end{cases} \tag{17}$$

where $[W_{(i,j)}]_{(m)}$ is the median value of then samples inside the window $W_{(i,j)}$. The sliding window $W_{(i,j)}$ is

$$W_{(i,j)} = \left\{X_{(i+r,j+r)} : r \in W\right\} \tag{18}$$

to be the window located at position i. The lower bound and upper bounds for relaxed median used in the experiments are 3 and 5 respectively.

When fourth order diffusion is applied to the images, the areas having small gradients are smoothed, and which having large gradients (edges and noise if any) remain undiffused and the blocky effects can be avoided. The gradients generated by noise can be subsequently removed by a relaxed median filter without affecting the image structure. However if the gradients are generated by edges, the relaxed median filter will not affect them. So as iteration continues, the nonlinear PDE removes the low level noise and subsequently the relaxed median filter removes the impulsive noise spikes. The proposed method preserves image structure much better than the other similar methods.

Figure 2 shows a comparative analysis of proposed method with the existing anisotropic median diffusion filter. The phantom molecular image used is same as in [9]. The image consists of five regions (background, cytoplasm, nucleus, mitochondria and endoplasmic reticulum). The noise distributions in different regions are also different. It can be seen that the *Ling-Bovik* method removes noise but generates false edges. Figure 2(d) shows that this artifact can be eliminated by using the proposed method. Figure 3 shows the plot of 54$^{\text{th}}$ row of the phantom image. The analysis shows that the proposed method preserves the curved structures in the image much better than its counterpart. Figure 4 illustrates the results of endothelial cells of rat aorta processed using anisotropic median diffusion filter and proposed filter. Close examination of the results shows that the artifacts generated by anisotropic median filter can be reduced by using the improved method.

4 Conclusion

A method to improve the performance of Ling-Bovik method is introduced in this paper. The artifacts generated by the Ling-Bovik method can be can be considerably reduced by this approach. The method preserves curve like structures and edges much better than the existing method. The filter is tested against molecular images and standard test images (by adding different levels of noise). The analysis shows that Ling-Bovik method can be improved by applying the proposed approach.

References

1. Yezzi, A.: Modified curvature motion for image smoothing and enhancement. IEEE Trans. Image Process. **7**, 345–352 (1998)
2. Hero, A., Krim, H.: Mathematical methods in imaging. IEEE Signal Process. Mag. **19**, 13–14 (2002)
3. Hamza, A.B., Krim, H.: Robust environmental image denoising. In: The ISI International Conference on Environmental Statistics and Health (2003)
4. Hamza, A.B., Krim, H., Unal, G.: Unifying probabilistic and variational estimation. IEEE Signal Process. Mag. **19**(5), 37–47 (2002)
5. Ben Hamza, A., Krim, H., Zerubia, J.: A nonlinear entropic variational model for image filtering. EURASIP J. Appl. Signal Process. **16**, 2408–2422 (2004)
6. Greer, J.B., Bertozzi, A.L.: Travelling wave solutions of fourth order PDEs for image processing. SIAM J. Math. Anal. **36**, 38–68 (2004)
7. Perona, P., Malik, J.: Scale-space and edge detection using anisotropic diffusion. IEEE Trans. Pattern Anal. Mach. Intel. **12**, 629–639 (1990)
8. Whitaker, R.T., Prizer, S.M.: A multi scale approach to nonuniform diffusion. CVGIP: Image Underst. **57**(1), 99–110 (1993)
9. Ling, J., Bovik, A.C.: Smoothing low-SNR molecular images via anisotropic median diffusion. IEEE Trans. Med. Imaging **21**, 377–384 (2002)
10. Rajan, J., Kaimal, M.R.: Image denoising using wavelet embedded anisotropic diffusion (WEAD). In: Proceedings of IET International Conference on Visual Information Engineering 2006, pp. 589–593, September 2006
11. Rajan, J., Kaimal, M.R.: Speckle reduction in images with WEAD & WECD. In: Computer Vision, Graphics and Image Processing. Lecture Notes in Computer Science (LNCS), vol. 4338, pp. 184–193. Springer, New York (2006)
12. Hamza, A.B., Escamilla, P.L., Aroza, J.M., Roldan, R.: Removing noise and preserving details with relaxed median filters. J. Math. Imaging Vis. **11**, 161–177 (1999)
13. Pitas, I., Venetsanopoulos, A.N.: Nonlinear Digital Filters: Principles and Applications. Kluwer Academic, Dordrecht (1990)
14. You, Y.L., Kaveh, M.: Fourth-order partial differential equations for noise removal. IEEE Trans. Image Process. **9**, 1723–1730 (2000)
15. Lysaker, M., Lundervold, A., Tai, X.C.: Noise removal using fourth-order partial differential equation with applications to medical magnetic resonance images in space and time. IEEE Trans. Image Process. **12**, 1579–1590 (2003)
16. Wei, G.W.: Generalized Perona-Malik equation for image processing. IEEE Signal Process. Lett. **6**, 165–167 (1999)
17. You, Y.L., Xu, W., Tannenbaum, A., Kaveh, M.: Behavioral analysis of anisotropic diffusion in image processing. IEEE Trans. Image Process. **5**, 1539–1553 (1996)
18. Mrazek, P., Weickert, J., Steidl, G.: Correspondence between wavelet shrinkage and nonlinear diffusion. In: Scale-Space 2003. LNCS, vol. 2695, pp. 101–116. Springer, New York (2003)

19. Lijima, T.: Basic theory on normalization of pattern. Bull. Electrotech. Lab. **26**, 368–388 (1962)
20. Charbonnier, P., Aubert, G., Feraud, L.B., Barlaud, M.: Two deterministic half-quadratic regularization algorithms for computed imaging. IEEE Int. Conf. Image Process. **2**, 168–172 (1994)
21. Weickert, J.: Anisotropic Diffusion in Image Processing. ECMI Series. Teubner, Stuttgart (1998)
22. Andreu, F., Ballester, C., Caselles, V., Mazn, J.M.: Minimizing total variation flow. Differ. Integral Equ. **14**, 321–360 (2001)
23. Keeling, S.L., Stollberger, R.: Nonlinear anisotropic diffusion filters for wide range edge sharpening. Inverse Probl. **18**, 175–190 (2002)
24. Rudin, L., Osher, S., Fatemi, E.: Nonlinear total variation based noise removal algorithms. Physics D **60**, 259–268 (1992)
25. Greer, J.B.: Fourth order diffusion for image processing. Ph.D. dissertation, Dept. of Mathematics, Duke University (2003)
26. Hamza, A.B., Krim, H.: Image denoising: a nonlinear robust statistical approach. IEEE Trans. Signal Process. **49**(12), 3045–3054 (2001)